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Corrigé TD (GB ABB/IAB 2019)

2- Simplifier les expressions :

• $x = \log_3 (81 \sqrt[3]{3})$

On peut écrire, de manière équivalente :

$$3^x = 81 \sqrt[3]{3} = 3^4 \cdot 3^{1/3} = 3^{4+1/3} = 3^{\frac{12+1}{3}} = 3^{13/3}$$

$$\Rightarrow \boxed{x = \frac{13}{3}}$$

• $x = \log_{1/3} (27 \sqrt{3})$

$$\left. \begin{aligned} \left(\frac{1}{3}\right)^x &= 27 \sqrt{3} = 3^3 \cdot 3^{1/2} = 3^{\frac{6+1}{2}} = 3^{7/2} \\ \text{"} \\ (3^{-1})^x &= 3^{-x} \end{aligned} \right\} \Rightarrow -x = \frac{7}{2} \Leftrightarrow \boxed{x = -\frac{7}{2}}$$

• $x = \log_{3\sqrt{3}} (81)$

$$\left. \begin{aligned} (3\sqrt{3})^x &= 81 = 3^4 \\ \text{"} \\ (3 \cdot 3^{1/2})^x &= (3^{1+1/2})^x = (3^{3/2})^x = 3^{\frac{3}{2}x} \end{aligned} \right\} \Rightarrow \frac{3}{2}x = 4 \Rightarrow \boxed{x = \frac{8}{3}}$$

• $x = \log_{2^3} (64)$

$$\left. \begin{aligned} (2^3)^x &= 64 = 2^6 \\ \text{"} \\ 2^{3x} & \end{aligned} \right\} \Rightarrow 3x = 6 \Rightarrow \boxed{x = 2}$$

- $x = \log_{1/3} (9\sqrt{3})$

$$\left. \begin{array}{l} \left(\frac{1}{3}\right)^x = 9\sqrt{3} = 3^2 \cdot 3^{1/2} = 3^{2+1/2} = 3^{5/2} \\ \text{"} \\ (3^{-1})^x = 3^{-x} \end{array} \right\} \Rightarrow -x = 5/2 \Rightarrow \boxed{x = -\frac{5}{2}}$$

- $x = \log_5 (125 \sqrt[3]{5})$

$$5^x = 125 \sqrt[3]{5} = 5^3 \cdot 5^{1/3} = 5^{3+1/3} = 5^{10/3}$$

$$\Rightarrow \boxed{x = \frac{10}{3}}$$

- $x = \log_{1/7} (49)$

$$\left. \begin{array}{l} \left(\frac{1}{7}\right)^x = 49 = 7^2 \\ \text{"} \\ (7^{-1})^x = 7^{-x} \end{array} \right\} \Rightarrow -x = 2 \Rightarrow \boxed{x = -2}$$

- $x = \log_{3\sqrt{3}} (81)$ (déjà fait plus haut, 3^{ème} exercice)