

An introduction to black hole spacetimes and gravitational radiation

(Lectures at the IMSP, Bénin)

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Abstract

In these lectures we present the basic geometric and physical elements of black hole spacetimes, in the context of the gravitational collapse picture provided by General Relativity. After a heuristic invitation to the subject, we synthetically present the needed mathematical elements of General Relativity, leading to the formulation of Einstein equations. We focus first on the vacuum spherically symmetric case characterised by the Schwarzschild solution, archetype for more generic black hole spacetimes. After presenting a choice of some relevant properties in spherical and axial symmetry, in particular the uniqueness of the Kerr solution, we sketch the generic dynamical picture of black hole spacetimes. We conclude with a discussion on gravitational radiation and its emission by compact sources, focusing on binary black holes. We conclude by briefly presenting the gravitational waveforms of the binary black hole events detected by Advanced LIGO, a benchmark in the study of black holes.

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 - 1.3 From tides to curvature.
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 - 2.2 Vector and tensor fields. Lie derivative.
 - 2.3 The metric tensor. Local isometries (Killing vectors). Example: Rindler “spacetime”.
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- 3.3 Curvature. The geodesic deviation equation. Einstein equations.
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