An introduction to black hole spacetimes and gravitational radiation

(Lectures at the IMSP, Bénin)

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Abstract

In these lectures we present the basic geometric and physical elements of black hole spacetimes, in the context of the gravitational collapse picture provided by General Relativity. After a heuristic invitation to the subject, we synthetically present the needed mathematical elements of General Relativity, leading to the formulation of Einstein equations. We focus first on the vacuum spherically symmetric case characterised by the Schwarzschild solution, archetype for more generic black hole spacetimes. After presenting a choice of some relevant properties in spherical and axial symmetry, in particular the uniqueness of the Kerr solution, we sketch the generic dynamical picture of black hole spacetimes. We conclude with a discussion on gravitational radiation and its emission by compact sources, focusing on binary black holes. We conclude by briefly presenting the gravitational waveforms of the binary black hole events detected by Advanced LIGO, a benchmark in the study of black holes.

Contents

3.3 Curvature. The geodesic deviation equation. Einstein equations.

4. **Lecture 4**: The Schwarzschild solution (1916-2016).

   4.1 Spherical symmetry. SAGE and Sagemanifols: computer algebra for manifolds.

   4.2 Schwarszschild solution and the Birkhoff theorem.

   4.3 Conformal compactification of Schwarzschild spacetime.

5. **Lecture 5**: Some topics on black holes: dynamics, rotation and uniqueness.

   5.1 Orbits around Schwarzschild. Examples of dynamical collapse.

   5.2 Gravitational redshift. Event horizon and apparent horizons.

   5.3 The Kerr solution. Rotation and frame dragging. Uniqueness.

6. **Lecture 6**: The generic black hole picture.

   6.1 Asymptotic flatness, horizons, trapped surfaces and singularity theorems.

   6.2 The initial value problem of General Relativity.

   6.3 The generic picture of gravitational collapse: theorems and conjectures.

7. **Lecture 7**: Gravitational waves (1916/1918-2016).

   7.1 Linearized Einstein equations.

   7.2 Effect of gravitational waves on test particles. Polarizations.

   7.2 Generation of gravitational waves: the quadrupole formula.

8. **Lecture 8**: Aspects of gravitational radiation from binary (black hole) systems.

   8.1 Radiation from a point mass: circular orbit and radial infall into a black hole.

   8.2 The binary black hole dynamical problem.

   8.3 Qualitative waveform of collapsing black hole binaries. The GW150914 event.