Lecture 1
Motivation to Black Hole physics

April 17, 2013

Abstract

General invitation to the course.

Contents

A natural starting point to the study of black holes is to consider the ultimate fate of stars of sufficiently high mass. This *gravitational collapse* approach is not the only possible avenue to the problem, but it has the virtue of providing a general framework that illustrates some of the main aspects, not only of black hole physics, but also of gravitational physics, this including in particular General Relativity. Moreover, it also follows the historical route to the topic.

1 The classical standard picture of gravitational collapse: a first physical overview

1.1 Star structure

We start by considering a simplified Newtonian description of stars. The structure of stars is basically governed by three simple laws, namely hydrostatic equilibrium, energy transport and

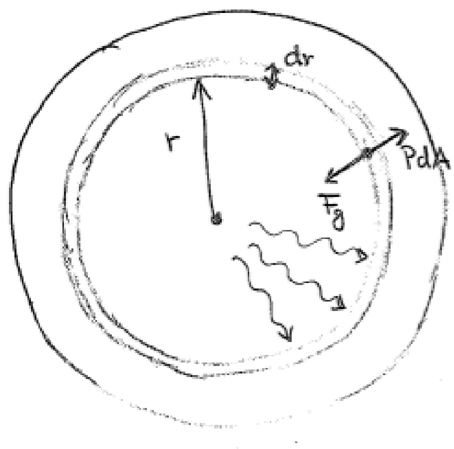

Figure 1: Star as a equilibrium between gravitational force and expanding pressure.

energy generation. For a spherical symmetric star (see Fig. 1.1):

$$
\begin{aligned}
\frac{dM(r)}{dr} &= 4\pi r^2 \rho(r) \\
\frac{dP(r)}{dr} &= -\frac{GM(r)}{r^2}\rho(r) \quad \text{(hydrostatic equilibrium)} \\
\frac{dL(r)}{dr} &= 4\pi r^2 \epsilon\, \rho(r) \quad \text{(energy conservation)} \\
\frac{dT(r)}{dr} &= -\frac{1}{4\pi r^2 \lambda} L(r) \quad \text{(energy transport)}
\end{aligned}
$$

where the primary variables of the system $M(r), P(r), L(r), T(r)$:

$M(r)$: mass contained from the center $r = 0$ to the shell of radius r
$P(r)$: pressure at radius r
$L(r)$: energy flow through the sphere of radius r
$T(r)$: temperature at radius r.

In order to close the system we need:

- Equation of state: $P = P(\rho, T, X_i)$, or inverting $\rho = \rho(P, T, X_i)$
- Coefficient of conductivity: $\lambda = \lambda(\rho, T, X_i)$
- Energy production rate: $\epsilon = \epsilon(\rho, T, X_i)$

with X_i accounting for the chemical composition. In addition we need boundary conditions. This would parametrize the stars in terms of its radius. However, the radius is a bad parameter since it is difficult to determine either experimentally or *a priori*. A better choice is to choose the mass of the star. For this we rewrite (1), with the mass contained inside a given shell as parameter:

$$
\begin{aligned}
\frac{dr(M)}{dM} &= \frac{1}{4\pi r^2 \rho(M)} \\
\frac{dP(M)}{dM} &= -\frac{GM}{4\pi r^4} \quad \text{(hydrostatic equilibrium)} \\
\frac{dL(M)}{dM} &= \epsilon \quad \text{(energy conservation)} \\
\frac{dT(M)}{dM} &= -\frac{1}{16\pi^2 r^4 \lambda \rho(M)} L(M) \quad \text{(energy transport)}
\end{aligned}
$$

Appropriate (approximate) boundary conditions are:

$$r(0) = 0 \ , \ L(0) = 0 \ , \ P(M_{\rm star}) = 0 \ , \ T(M_{\rm star}) = 0$$

where $M_{\rm star}$ is the total mass of the star, which becomes a parameter in the model.

The crucial ingredients to counteract the gravity and keep hydrostatic equilibrium are the energy production rate and the equation of state. In gravitational collapse, part of the initial gravitational energy is used to heat the matter. However, the resulting increase in the pressure is not enough to reach the hydrostatic equilibrium. When the temperature is high enough nuclear reactions are initiated and the resulting ϵ is able to keep the equilibrium and the life of star is span. However, once this nuclear fuel is exhausted, the hydrostatic equilibrium is once more lost and collapse continues. The collapse continues until matter reaches an stage in which the equation of state is *rigid enough*. This leads to the formation of compact stars.

1.2 Compact stars

Degenerate Fermi gas. Fermions satisfy Pauli's exclusion principle, that prevents two fermionic particles to be in the same quantum state. Electrons, protons and neutrons are fermionic particle of spin 1/2. This in particular means that for a given momentum p there can only be two particles (spin-up and spin-down). As a consequence, particles occupy the phase space till a maximum Fermi momentum $p_{\rm F}$. As a consequence of this motion, the resulting *degenerate Fermi gas* acquires a pressure. It is this pressure that balances the gravitational force.

In our context the relevant particles are electrons and neutrons since, at sufficiently high densities, protons and electrons suffer a weak force process (a form of beta-decay) known as *neutronization*:

$$e^- + p^+ \to n^0 + \nu_e \tag{1}$$

The equation of state of a degenerate Fermi gas has two different regimes: i) non-relativistic regime, when the reached Fermi momentum satisfy $p_{\rm F} \ll mc$ and ii) the ultra-relativistic regime, when $p_{\rm F} \gg mc$. The equations of state differ in both cases, although they share the key feature of not depending on the temperature. We have (see e.g. [1])

$$\begin{array}{ll} \text{relativistic Fermi gas:} & P = K\frac{\hbar^2}{m}(\frac{N}{V})^{\frac{5}{3}} \\ \text{ultra-relativistic Fermi gas:} & P = K'\hbar c(\frac{N}{V})^{\frac{4}{3}} \end{array} \tag{2}$$

where N is the total number of fermions and K and K' are dimensionless constants.

Degenerate stars. Estimating the density as $\rho \sim M/R^3$ and the pressure gradient as $\nabla P \sim P/R$ we can write the hydrostatic equilibrium equation as $\frac{GM\rho}{R^2} \sim \nabla P$

$$GM^2 \sim PR^4 \tag{3}$$

We also introduce the mass per Fermi particle $m' = M/N$. Then, we can write:

- *Non-relativistic regime*: From Eq. (2)

$$P \sim \frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{\frac{5}{3}} \sim \frac{\hbar^2}{m}\cdot\frac{N^{\frac{5}{3}}}{R^5} \tag{4}$$

so that from (3) we have

$$GM^2 \sim \frac{\hbar^2}{m} \cdot \frac{N^{\frac{5}{3}}}{R} \tag{5}$$

and using m'

$$R \sim \frac{\hbar^2}{Gmm'^{5/3}} \frac{1}{M^{1/3}} \tag{6}$$

From this we conclude that the larger the mass, the smaller the radius. This is the crucial ingredient of the Fermi degenerate equation of state. It implies that as we consider increasing masses the density and pressure also grow until we reach a (ultra-)relativistic regime for the Fermi gas.

- *Ultra-relativistic regime*: Repeating the steps:

$$P \sim \hbar c \left(\frac{N}{V}\right)^{\frac{4}{3}} \sim \hbar c \cdot \frac{N^{\frac{4}{3}}}{R^4} \tag{7}$$

and

$$GM^2 \sim \hbar c \cdot N^{\frac{4}{3}} \tag{8}$$

Remarkably, the radius disappears from the equilibrium relation, so that the mass is fixed

$$M \sim M_\star = \frac{(\hbar c/G)^{3/2}}{m'^2} \tag{9}$$

The conclusion is that for masses below $M_\star$, the pressure associated with the degenerate Fermi gas supports the gravitational force. As the mass increases the radius decreases and the fermions become more and more relativistic. Then the ultra-relativistic regime provides the critical mass mass that can be supported by this mechanism.

White dwarfs are compact stars in which the degenerate Fermi gas is composed of electrons. In this case, the limit to the mass is known as the *Chandrasekhar limit* and is about $1.44M_\odot$. For neutron stars, resulting from supernova core-collapses of massive stars, the limit is referred to as *Tolman-Oppenheimer-Volkoff* and is less precisely established, depending essentially on the details of the equation of state. A particular (exotic) class of *neutron* star are *quark stars* in which the relevant degenerate fermions are *strange stars* (postulated as the ground state of baryonic matter).

Beyond this mass, no mechanism is known capable of stopping the gravitational collapse. The eventual result of this process is what we know as *black hole*. Black holes are a dramatic extreme case of a characteristic feature of General Relativity: *bending of light*. And the latter is a manifestation of a more general concept: spacetime curvature. Let us explore how this concept emerges in the study of gravitation.

2 A first glimpse into to Gravity as spacetime curvature

Special relativity offers a description of relativistic motion in the case that gravity can be neglected. However, compact stars in the last stages of gravitational collapse involve both relativistic motion and strong gravitational fields. In this section we describe the tension existing

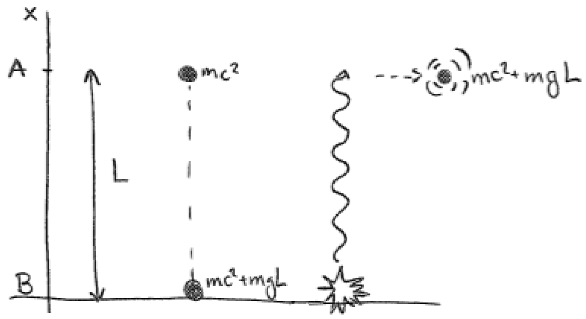

Figure 2: Violation of energy conservation if photons are not gravitationally redshifted.

between special relativity and the incorporation of gravity in the picture, ultimately leading to the notion of a curved spacetime:

$$\left.\begin{array}{c}\text{Special Relativity}\\\text{Gravity}\end{array}\right\}\text{Tension}\longrightarrow\text{Spacetime curvature}$$

The main line of reasoning is that the marriage between light propagation and gravity implies the existence of a gravitational redshift effect, and that the latter is incompatible with special relativity, leading to the notion of an intrinsically curved spacetime:

$$\left.\begin{array}{c}\text{Gravitational Redshift}\\\text{flat spacetime}\end{array}\right\}\longrightarrow\text{Spacetime curvature}$$

We follow essentially the discussion in [5].

2.1 Gravitational redshift from energy conservation

Let us start by reviewing the original Einstein argument, based on physical reasoning (namely energy conservation), leading to the existence of gravitational redshift.

We dwell here in a Newtonian description of gravity. Let us consider a particle of mass m at a height L in a constant gravitational field (with gravitational acceleration g).

i) Initially the particle is at A and its rest energy is:

$$E^A = mc^2 \quad . \tag{10}$$

ii) It falls to B, having a rest plus kinetic energy:

$$E^B = mc^2 + mgL \quad . \tag{11}$$

iii) At B, the particle is annihilated producing a photon with (the same) energy:

$$E^B_{ph} = mc^2 + mgL \quad . \tag{12}$$

Then the photon goes back upwards to A. If the energy of the photon at A were $E^A_{ph} = E^B_{ph} = mc^2 + mgL$, then we are able to *create* energy that we can use. Indeed, the photon at A can be transformed into a particle of mass m and some additional energy (thermal, kinetic...) (see Fig. 2.1):

$$E^A = mc^2 + mgL \quad . \tag{13}$$

We can repeat the process n times after which we have at A a particle of mass m and a production of extra energy

$$E^A = mc^2 + nmgL \quad , \tag{14}$$

producing an arbitrarily large violation of the energy conservation.

The way out is to accept that the photon loses energy when going from B to A: the photon has to climb the gravity potential exactly as a massive particle would have to. Therefore starting from B with an energy E^B_{ph} it arrives at A with an energy E^A_{ph}

$$E^B_{ph} = mc^2 + mgL = mc^2(1 + \frac{gL}{c^2}) \quad \rightarrow \quad E^A_{ph} = mc^2 \tag{15}$$

Now Einstein's argument incorporates another piece of physical reasoning. In particular, at this point one uses the relation between energy of a photon and its wavelength given by quantum theory, namely

$$E_{ph} = h\nu = \hbar\omega \quad . \tag{16}$$

Then, using $\lambda = c/\nu$ and the redshift factor z introduced as

$$z = \frac{\lambda_A - \lambda_B}{\lambda_B} \quad , \quad 1 + z = \frac{\lambda_A}{\lambda_B} \quad , \tag{17}$$

one gets

$$1 + z = \frac{\lambda_A}{\lambda_B} = \frac{\nu_B}{\nu_A} = \frac{h\nu_B}{h\nu_A} = \frac{E_B}{E_A} = (1 + \frac{gL}{c^2}) \quad , \tag{18}$$

so that

$$z = \frac{gL}{c^2} \quad . \tag{19}$$

This expression for the redshift of a photon "going up" a gravitational field, *deduced* by Einstein in 1911 using this chain of heuristic physical arguments, would be experimentally confirmed only in 1959 by Pound & Rebka [7].

2.2 Gravitational redshift and the principle of equivalence

The previous discussion of the gravitational redshift is physically inspiring, but can be criticized on consistency grounds. The discussion can be recast in a more systematic ("first-principles") form in terms of the key ingredient in the process of the *geometrization* of the gravitational field: the *equivalence principle*. In its more basic form it states:

"All effects of a uniform gravitational field are identical
to the effects of a uniform acceleration of the coordinate system."

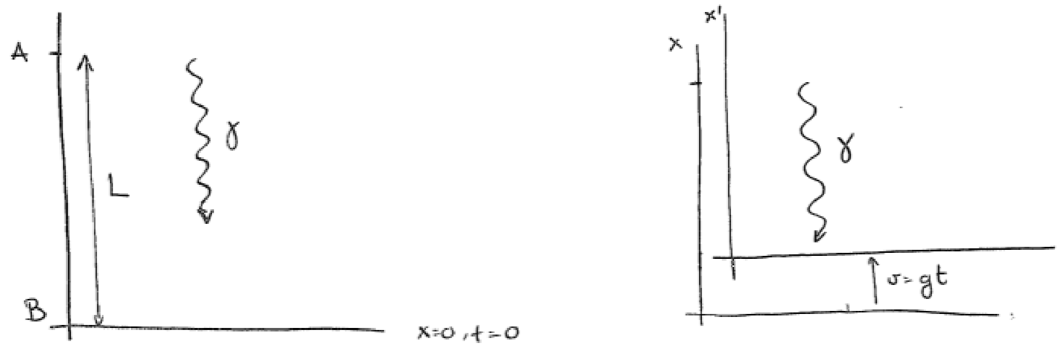

Figure 3: Photon moving in a constant gravitational field or, equivalently, in an accelerated frame.

This is a generalization of the simple remark in the context of Newtonian particle dynamics, where we can write

$$m\frac{d^2x}{dt^2} = F = mg \quad \Longleftrightarrow \frac{d^2x}{dt^2} - g = 0 \; ; \frac{d^2x'}{dt^2} = 0 \quad , \tag{20}$$

with

$$x' = x - \frac{1}{2}gt^2 = x + \frac{1}{2}at^2 \; ; \; a = -g \quad . \tag{21}$$

As in the case of the *relativity principle* leading to special relativity, the key element here is the extension of the validity of the statement to ALL possible effects, this including electromagnetic ones, in particular light propagation.

Let us consider again the points A and B above, standing in a constant gravitational field. A photon γ is emitted from A to B.

According to the equivalence principle we can consider the photon suffering an acceleration $a = g$, as in an accelerated rocket in absence of gravitational field[1]. See Fig.2.2

Described in the non-accelerated frame, points A and B move in a uniformly accelerated motion as

$$x_A = L + \frac{1}{2}gt^2 \quad ; \quad x_B = \frac{1}{2}gt^2 \tag{22}$$

i) The photon is sent from A at $t = 0$, so that B receives it at $t = t_1$. The traveled distance is

$$x_A(0) - x_B(t_1) = ct_1 \quad , \quad L - \frac{1}{2}gt_1^2 = ct_1 \; . \tag{23}$$

ii) A second photon (or the next crest in a wave train) of is sent from A at $t = \Delta\tau_A$ and B receives it a time $\Delta\tau_B$ after receiving the first photon, that is at $t_2 = t_1 + \Delta\tau_B$. The distance traveled by the second photon is

$$x_A(\Delta\tau_A) - x_B(t_2) = c(t_2 - \Delta\tau_A) = c(t_1 + \Delta\tau_B - \Delta\tau_A) \quad . \tag{24}$$

[1]We will neglect second order terms (such as $\left(\frac{v}{c}\right)^2$ or $\left(\frac{gL}{c^2}\right)^2$) in the following discussion.

The left hand side can be re-expressed as

$$x_A(\Delta\tau_A) - x_B(t_2) = x_A(\Delta\tau_A) - x_B(t_1 + \Delta\tau_B) = L + \frac{1}{2}g(\Delta\tau_A)^2 - \frac{1}{2}g(t_1 + \Delta\tau_B)^2$$
$$= L + \frac{1}{2}g(\Delta\tau_A)^2 - \frac{1}{2}gt_1^2 - gt_1\Delta\tau_B - \frac{1}{2}g\tau_B^2 \approx L - \frac{1}{2}gt_1^2 - gt_1\Delta\tau_B \quad , \tag{25}$$

where we have neglected second-order terms in $\Delta\tau$'s. That is

$$L - \frac{1}{2}gt_1^2 - gt_1\Delta\tau_B \approx c(t_1 + \Delta\tau_B - \Delta\tau_A) \quad . \tag{26}$$

Subtracting (23) from (26) we get

$$-gt_1\Delta\tau_B = c(\Delta\tau_B - \Delta\tau_A) \Leftrightarrow \Delta\tau_A = \Delta\tau_B(1 + \frac{gt_1}{c}) \quad . \tag{27}$$

Finally, approximating at first order from (23), $t_1 \approx \frac{L}{c}$ we get

$$\Delta\tau_A = \Delta\tau_B(1 + \frac{gL}{c^2}) \quad . \tag{28}$$

iii) Now, expressing the time intervals $\Delta\tau$'s in terms of frequencies, $\Delta\tau = 1/\nu$ we write

$$\nu_B = \nu_A(1 + \frac{gL}{c^2}) \quad , \tag{29}$$

from where, again

$$1 + z = \frac{\lambda_A}{\lambda_B} = \frac{\nu_B}{\nu_A} = (1 + \frac{gL}{c^2}) \quad , \tag{30}$$

and

$$z = \frac{gL}{c^2} \quad . \tag{31}$$

as in Eq. (19).

2.3 Gravitational redshift implies curvature of spacetime

The previous discussions have led us to the notion that light propagating in a gravitational field gets redshifted. We can accept this either from Einstein's physical argument, or as a consequence of the equivalence principle, or simply as an experimental fact from Pound & Rebka experiment.

On the other hand, special relativity has already shown that a consistent description of particle kinematics and electrodynamics involves a spacetime perspective on space and time. Space and time are recast in a single geometric structure modeled as a linear space endowed with a flat metric of Lorentzian type: the Minkowski spacetime. At this point we show, following an argument of Schild (see Fig. 2.3), that the presence of a gravitational redshift is incompatible with the existence of a *flat* spacetime like in special relativity. Schild's argument is independent of the detailed mathematical description of the gravitational field. Only stationarity plays a key role in the argument. Let us consider two observers A and B at rest one with respect to the other and with respect to the Earth (namely, the source of the gravitational field). Whatever the nature of the gravitational field is, it will present a stationary configuration.

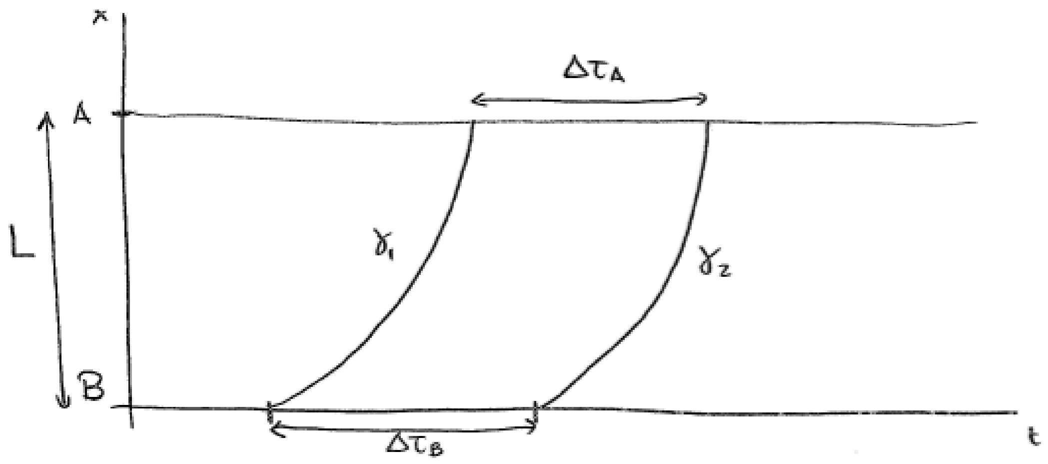

Figure 4: Diagram for Schild's argument on the incompatibility between gravitationa redshift and flat spacetime.

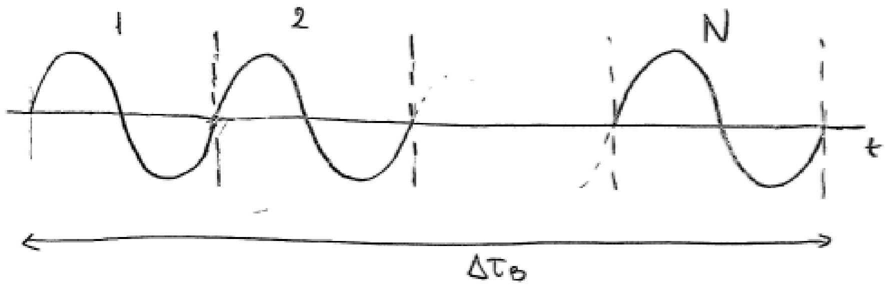

Figure 5: Wave train of signals emitted from B towards A.

At some given time, a signal is emitted from B towards A. Let us assume that it is a periodic signal with N cycles. Then (see Fig. 2.3)

$$N = \nu_B \Delta\tau_B \quad , \tag{32}$$

with ν_B the frequency and $\Delta\tau_B$ the elapsed time of the signal.

The receiver at A receives the N cycles in a time $\Delta\tau_A$, so that

$$N = \nu_A \Delta\tau_A \quad , \tag{33}$$

and

$$\nu_A \Delta\tau_A = \nu_B \Delta\tau_B \quad . \tag{34}$$

According to previous discussions, if a redshift is present we have $\nu_B > \nu_A$ and, as a consequence

$$\Delta\tau_A > \Delta\tau_B \quad . \tag{35}$$

However, since the gravitational field is static and the observers do not move, trajectories γ_1 and γ_2 of the respective photons must be congruent curves, i.e. γ_1 and γ_2 are the sames curves except from their *positions* in space. If such curves are placed in a *flat* space and time diagram (namely, the spacetime), they must form a parallelogram, so that

$$\Delta\tau_A = \Delta\tau_B \quad , \tag{36}$$

in contradiction with (35). This contradiction indicates that the flat spacetime of special relativity, namely Minkowski spacetime, is not adequate for the description of gravity. If we want to stick to the spacetime vision of space and time provided by special relativity, then we must renounce to spacetime flatness. In particular, parallel light trajectories can start converging and diverging, in general *bending* in a curved spacetime. More generally, in this geometric spacetime perspective the presence of a gravitational field is realised through the curvature of spacetime. General Relativity provides a definite self-consistent manner of introducing physical sources to this spacetime curvature, namely through energy and stress of matter. At the same time, it endows this spacetime curvature, namely the gravitational field, with specific dynamics. We will address that in Lecture 4, when we describe Einstein equations.

3 Classical collapse: standard relativistic paradigm

As discussed above, a characteristic feature of General Relativity and, more generally of theories modeled on curved spacetimes, is the *bending* of light. Black holes constitute a dramatic extreme in which the light bending is so strong that it cannot leave a certain compact region of the space.

Let us give a brief overview of the current standard picture of classical gravitational collapse, that constitutes what one might call the *establishment picture of gravitational collapse*. This consists in a heuristic chain of theorems and conjectures providing a general conceptual framework:

i) *Singularity theorems* (Theorem). If enough *energy* is placed in a sufficiently compact region, so that light bending forces the local convergence of all emitted light rays and so-called "trapped surfaces" are formed, then a singularity develops in spacetime [6, 3, 4, 2].

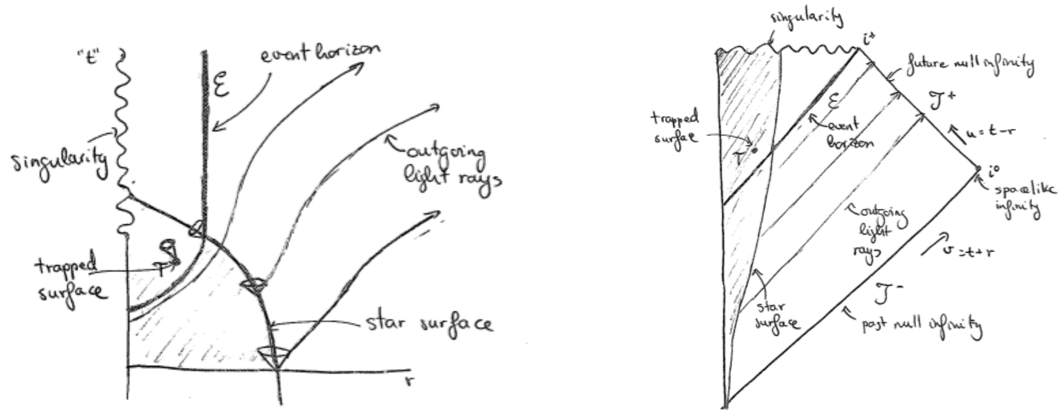

Figure 6: Establishment picture of gravitational collapse. The picture in the right is a Carter-Penrose spacetime diagram where lightlike rays lay at $\pm 45^o$. The thick line at 45^o line represents the event horizon, separating the black hole region to its left (containing the spacetime singularity corresponding to the horizontal oscillating line) from the rest of the spacetime.

ii) *(Weak) Cosmic Censorship* (Conjecture). In order to keep the predictability of the theory, the formed singularity should be hidden from a distant observer behind a so-called "event horizon", giving rise to a black hole region.

iii) *Spacetime stability* (Conjecture). If general relativity is a physically consistent theory of gravity, it is natural to expect that a system with a finite amount of energy must be eventually driven dynamically to stationarity. This is again a conjecture, now about the stability of a black hole spacetime.

iv) *Black hole uniqueness* (Theorems). The eventual stationary state is completely characterized by the mass and angular momentum of a the resulting (Kerr) black hole. This is usually referred to as the *no-hair* property of stationary black holes.

The *establishment* picture provides a general systematic framework for posing and addressing issues related to black hole spacetimes. In particular it provides a working program to the study of many of the key aspects to General Relativity. On the other hand, it must be said that nearly every single aspect of it is challenged at one place or another in gravitational physics. In quite a literal sense, the goal of this course is to explain the diagram in Figure 3.

4 Interest in Black Hole physics

Why should done study black holes? A straightforward valid astrophysical answer could be, simply, because they are out there. Although this is indeed a valid answer, this does not make justice to the richness of the subject. Black holes indeed constitute, on the one hand, crucial ingredients for the understanding of astrophysical and cosmological processes. But, on the other hand, they also provide clues for the understanding of fundamental issues in the theory as well as a cornerstone in modern developments in theoretical physics.

4.1 Black holes in astrophysics and Cosmology

4.1.1 Compacity parameter

By now we have a general broad picture of the destiny of star attending to its final mass. The resulting final stage is a compact massive object, namely white dwarf stars, neutron stars or black holes. One might expect that the key parameter controlling the transition from white dwarfs to black hokes to be the density of the final object, but this not quite so. Indeed the (formal) density of supermassive black hole can be indeed very small. The relevant parameter is the one controlling the ability of emitted light rays to escape from the object, and this is controlled by a dimensionless parameter Ξ referred to as the *compacity parameter*

$$\Xi = \frac{GM}{c^2 R} \quad , \tag{37}$$

where M is the mass of the object and R is its characteristic scale (radius). In order to gain a qualitative intuition of why the radius enters with as R^{-1}, and not as R^{-3} as it would be the case for a density, it is enough to consider the Newtonian description of the escape velocity. For this we consider a particle of mass m emitted with velocity v from the surface of an spherical object of mass M at radius R. Its total energy is $E_R = \frac{1}{2}mv^2 - \frac{GMm}{R}$. The escape velocity is the one that permits the particle to reach an infinity distance with vanishing velocity, so that $E_\infty = 0$. Conservation of energy then gives

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 \Leftrightarrow \frac{1}{2}v^2 = \frac{GM}{R} \quad . \tag{38}$$

Considering the existence of maximum velocity $v = c$, for radius $R < \frac{2GM}{c^2}$ no particle can escape to infinity (this argument was presented already by Michell and Laplace). In other words, for a spherical object if the rate $\frac{GM}{c^2 R}$ is larger than $\frac{1}{2}$ no light can escape. Remarkably, this estimation in Newtonian theory results to be exact when revisited in the context of General Relativity, as we will see in Lecture 5. This justifies the use of (37) as the relevant parameter in this context. We provide

Object	M ($M_\odot$)	R (km)	Density (kg/m^3)	Ξ
Earth	3×10^{-6}	6×10^3	5×10^3	10^{-10}
Sun	1	7×10^5	10^3	10^{-6}
White Dwarf	$\sim 0.1 - 1.4$	$\sim 10^4$	10^{10}	$10^{-4} - 10^{-3}$
Neutron Star	$\sim 1 - 3$	~ 10	10^{18}	0.2
Stellar Black Hole (spherical)	$>\sim 3$	$9(M = 3M_\odot)$	-	0.5
Stellar Black Hole (extremal)	$>\sim 3$	$4.5(M = 3M_\odot)$	-	1
Massive Black Hole	$\sim 10^9$	$20U.A.$	-	$0.5 - 1$

4.1.2 Types of black holes

Attending to their mass we can classify black holes in different types:

i) Stellar mass black holes: $M \sim 3 - 30M_\odot$.

   These black holes are predicted by the gravitational collapse description discussed above, starting from highly massive stars. In this sense, they were predicted by the theory.

ii) Massive and supermassive black holes: $M \sim 10^5 - 10^9 M_\odot$.

Black hole of these masses came as a surprise from the need to explain the sources of energy associated with quasars (*quasi-stellar objects*). These are objects at very far distances emitting enormous amounts of energy and finally identified with active galactic nuclei emitting in X-ray, ultraviolet and radio. The emission is around three orders of magnitude that of the total optical luminosity of the parent galaxy. Supermassive black holes at the center of the galaxy offer a mechanism for the generation of such amounts of energy, at the expense of their huge gravitational energy. Along the years the black hole paradigm has become established in the understanding of the properties and evolutions of galaxies.

iii) Intermediate mass black holes: $M \sim 10^3 M_\odot$

There is no unambiguous evidence of the existence of black hole with these masses. They can play an important role in certain astrophysical processes and could be natural intermediate stages between stellar and massive black holes. However there is no observational evidence of their existence.

iv) Primordial black holes: mass up to $\sim 1 M_\odot$.

These are hypothetical black holes formed at early stages in the cosmological evolution of the Universe from the collapse of overdense matter regions. They could play an important role to explain the formation of cosmological structures in the Universe.

4.1.3 Evidence of black holes

i) *Stellar black holes.* Best candidates for stellar black holes are in binaries in which the companion is a normal (non-compact star) providing a flow of material into the black hole. Such material is heated as it forms an accretion disc, emitting in X-rays. From the determination of the orbital parameters one can infer the mass of *dark* object. If the mass is over $3M_\odot$ is a candidate for a black hole and one aims to refine the assessment as a black hole. For this, one can try to identify some of the signatures about the black hole presence provided by general relativity, e.g. i) absence of a rigid boundary surface, existence of an *innermost stable circular orbit* (see Lecture 9) affecting the properties of matter accretion discs, broadening of the $FeK\alpha$ line by gravitational redshift, characteristic distribution of mass and rotation multipoles...

See Table 1.1 in [1] for the best known 22 candidates. These studies, together with evolutionary models and observation of massive stars indicates that stellar black holes are actually very common objects. In our galaxy, the Milky Way, they are estimated to be around $10^8 - 10^9$, something corresponding to a fraction around $10^{-2} - 10^{-3}$ of the total number of stars (around 10^{11} in the Galaxy).

From an astrophysical point of view, stellar mass black holes are important ingredients in the explanation of jet structure of so-called micro-quasars or in models of (long) γ-ray bursts.

ii) *Massive black holes.* Although the mechanism of formation of these black holes is not known, massive and supermassive black holes stand as key ingredients in the most probable explanation of the galactic nuclei activity.

These black holes are at the core of the mechanism for the emission of relativistic jets. They are also able to provoke the tidal disruption of non-compact stars falling onto them and showing a characteristic flares in the electromagnetic spectrum. Maser radiation from quasars also opens a tools to measure parameters of black holes. Finally, it is worthwhile to note that quite recent observations of individual stars of the galactic center of the Milky Way (namely $SgrA*$) have permitted to establish the mass of the black hole at the center of our Galaxy as $4.6 \cdot 10^6 M_\odot$. The tools are very similar to the ones employed in the determination of the mass from the kinematics of binary systems.

Before ending this subsection we note that black holes in general, and binary black holes in particular, stand among the most important sources of gravitational radiation (see Lecture 11). The gravitational radiation emitted from the surrounding of a black hole portrays very characteristic signals of the dynamical spacetime geometry. In this sense, the ultimate tool to identify a compact object as a black hole is provided precisely from the radiation made of the same fabric as black holes: spacetime dynamics.

4.1.4 Black holes as *basic* objects in General Relativity

Black holes are not only relevant because of the role in some of the most violent events in the Universe in astrophysical and cosmological scenarios. They are objects of enormous theoretical interest on their own: on the one hand they represent particularly simple and clean probes into the strong-field regime of general relativity, and on the other hand they stand as a cornerstone piece in the puzzle of bringing together physics at different level of description, namely gravity, quantum mechanics and thermodynamics.

We simply list here some of the relevant aspects of black holes at a theoretical level:

- *Simple classical objects.* Black holes are simple strong gravity solutions in General Relativity. In fact, due to the "no-hair" theorems, in stationarity they are so simple that they can be described only and completely by two parameters. This is extraordinarily singular for a macroscopic object.

- *Two-body problem in general relativity.* Given that general relativity deals essentially with extended objects, the resolution of the motion problem is a very complicated problem by itself, that becomes only more complicate if we add the complexity associated to matter structure. In this sense, black holes provide a particularly clean "equation of state" to study in particular the binary problem in general relativity without having to bother simultaneously with hydrodynamical, rather than gravitational dynamics.

- *Probes into general relativity strong-field regime.* General relativity is well tested in the regime of weak gravitational fields, in particular through the dynamics of binary pulsars. However, the dynamics of the strong field regime and in particular the control and understanding of the decay properties of fields propagating in a strongly dynamical spacetime are poorly understood. Black holes provide a particularly suited probe to study both the stationary and dynamical aspects of the classical gravitational field.

- *Black hole thermodynamics.* The application of general relativity to black hole dynamics leads to a series of laws in perfect analogy with those of thermodynamics. The analogy reached a sounder physical status after the understanding by Hawking that a black hole actually radiate energy according to the black body spectrum of an object in thermal equilibrium, when semiclassical corrections are taking into account. This thermodynamical-like result stands as a solid prediction of the interplay between gravity and quantum mechanics and offers a test for any theory attempting to develop a quantum description of gravity.

- *Cornerstone at the gravity, quantum mechanics and thermodynamical crossroad.* The statistical mechanics understanding of the entropy of a black hole in terms of the number of states of the underlying system, is one of the most important task in approaches to quantum gravity. It offers a test, but also insight to develop avenues into the problem of marrying gravity and quantum mechanics. On the other hand, the evaporation of the black hole through Hawking radiation raises the issue of the unitarity of the black hole evolution description, leading to the black hole *information loss* problem.

- *Black holes in higher dimensions.* Motivated by quantum gravity scenarios involving higher spacetime dimensions (namely *string theory*), there is an interest in understanding classical solutions in higher dimensions presenting an event horizon. First, the uniqueness results associated with the "no hair" property of black hole is lost, offering a more complex panorama. Second, so-called *micro black holes* of up to $\sim 1M_\odot$ appear in speculative theories inspired in so-called *brane worlds.* Third, unexpected mathematical properties shared with four-dimensional black holes are maintained (namely the so-called *hidden-symmetries*), calling for a still missing explanation.

5 Summary of Lecture 1

1. Gravitational collapse and mass:
   i) Compact stars: radius decreases with mass.
   ii) Maximal mass for white dwarfs and neutron stars.
   iii) No known mechanism to stop the collapse above $\sim 3M_\odot$.

2. Black holes as a dramatic extreme case of light bending:
   i) Tension: Special Relativity AND Gravity.
   ii) Gravitational Redshift: incompatibility with flat spacetime.
   iii) Spacetime curvature: bending of light.

3. Standard picture of classical gravitational collapse:
   i) Chain of theorems and conjectures.
   ii) A conceptual framework for black holes (...and a "Course Program").
   iii) Every point in the framework is challenged.

4. Interest in Black Holes:
   i) Astrophysical and Cosmological.
   ii) Clean probe into the structure of the gravitational theory: General Relativity.
   iii) A key to physics unification and to new physics.

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