2018/19

Master Mathematical Physics (Dijon) - General Relativity
Exam (second session)
14 June (3 hours)

1. Given the Rindler spacetime with metric defined on $\mathbb{R} \times \mathbb{R}^+$ by the line element

$$ds^2 = -x^2 dt^2 + dx^2 \quad , \quad -\infty < t < \infty \ , \ 0 < x < \infty \ , \tag{1}$$

   i) Show that the Rindler spacetime can be seen as an open "wedge" of the Minkowski spacetime.

   ii) Obtain the Riemann tensor, Ricci tensor and Ricci scalar of the line element (1) and verify that they satisfy the vacuum Einstein equations.

2. Derive the expression for the gravitational redshift around a spherically symmetric massive body. That is, derive the relation between the frequence ν_A of a signal emitted by a stationary observer at a location r_A with respect to the massive body and the frequence ν_B received by a stationary observer at a location r_B (with $r_A < r_B$ and both r_A and r_B larger than the radius of the massive body).

3. i) The spacetime metric around a spherically symmetric planet is given by the Schwarzschild metric. We consider two astronauts in a spaceship orbiting the planet at a certain height. At a given time, one of them goes down (fastly, in a time scale δT_1) to the planet and stays there for a long time T (namely $T \gg \delta T_1$). After this time, he comes back (again fastly, δT_2 with $T \gg \delta T_2$) to the spaceship and meets the astronaut who stayed there. Justify which astronaut is younger at the moment of meeting again.

   ii) Given the metric on $\mathbb{R}^3$:

$$\boldsymbol{g} = f^2(z)\,(dx \otimes dx + dy \otimes dy) + dz \otimes dz \ ,$$

   with $f(z) > 0$, justify that $\boldsymbol{g}$ is conformally flat.